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1978 J. Phys. A: Math. Gen. 11 1687

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J. Phys. A: Math. Gen. Vol. 11, No. 8, 1978. Printed in Great Britain

On radiation pressure forces in cold magnetised plasma

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Received 9 January 1978

Abstract. Hydrodynamic equations governing the time-averaged motions of plasma electrons and ions in the presence of oscillating and steady electromagnetic fields are derived. In addition to well known gradient and collisional terms, new forces arise owing to the time dependence of the field amplitudes. Time-averaged particle flux densities are given.

1. Introduction

The problem of time-averaged forces (see Landau and Lifshitz 1957) exerted by oscillating electromagnetic fields on charged particles has been considered by many authors. Boot *et al* (1958), Gaponov and Miller (1958) and Weibel (1958) first derived the 'effective potential' proportional to the square of the electric field amplitude, which governs the time-averaged motion of particles. That result has been generalized in a series of subsequent papers, of which we refer at least to Pitaevskii (1960), Johnston (1960), Golovanivskii and Kuzovnikov (1961), Hora (1969) and Canobbio (1969). Further references together with a survey of results can be found in reviews by Motz and Watson (1967) and by Gorbunov (1973). In the papers quoted above, the time-averaged forces arise owing to the space dependence of field components. They are proportional to $\hat{E}^2/(|\boldsymbol{k}_0|a)$, where $\hat{E}$ is the wave electric field amplitude, $\boldsymbol{k}_0$ is the wavevector and a is the scale length of the field inhomogeneity. In a collisional plasma, forces proportional to $\hat{E}^2\nu/\omega$ arise, ν and ω being the collisional and wave frequencies (Johnston 1960).

For the case of non-magnetised plasma, it has been shown by Kadomtsev (1964), Fainberg and Shapiro (1965), Best (1968), Jungwirth (1972) and Klíma (1972) that the time dependence of the wave field amplitude may produce a force proportional to $\hat{E}^2/(\omega\tau)$, where $\tau = (\partial \ln \hat{E}^2/\partial t)^{-1}$. The results of Klíma (1972) have been extended to magnetised plasma for two particular cases of waves by Milantiev (1976). Washimi (1973) has pointed out the importance of such forces for the self-focusing of transverse waves propagating along the steady magnetic field. The general case of a dispersive magnetised medium has been analysed by Washimi and Karpman (1976). We shall return to their results later on.

Up to now, no general explicit expressions for time-averaged forces exerted separately on plasma electrons and ions by the oscillating field with time-dependent amplitude have been derived. Such expressions are necessary for finding the (time-averaged) current density and, consequently, for closing the set of time-averaged hydrodynamic and field equations. More rigorously, full equations of time-averaged motions of electrons and ions are to be derived.

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The purpose of the present paper is to deduce those equations for the case of cold magnetised plasma. In § 2, basic equations and assumptions are formulated. A hydrodynamic equation of motion which includes the forces proportional to $\hat{E}^2/(\omega\tau)$ is derived in § 3. It turns out that it is necessary to average the hydrodynamic velocity along the oscillating trajectory of the fluid volume element (§ 4). The final equation (5.6) of the time-averaged motion is given in § 5 together with expressions for the (time-averaged) electric current density. Section 6 compares the present analysis with relevant previous results.

2. Basic equations and assumptions

We assume the presence of steady fields $\boldsymbol{E}_0$ and $\boldsymbol{B}_0$ together with an oscillating electromagnetic field $\boldsymbol{\mathscr{E}}$ and $\boldsymbol{\mathscr{H}}$. In Cartesian coordinates x_i, $i = 1, 2, 3$,

$$\mathscr{E}_i = \hat{E}_i(t, \boldsymbol{x}) \cos(\omega t - k_{0j}x_j + \alpha_i), \tag{2.1}$$

where $\hat{E}_i$ and α_i are the slowly varying (real) amplitude and phase shift. The hydrodynamic velocity $\boldsymbol{v}$ of cold plasma particles is governed by the following equation:

$$\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} = \frac{e}{m}\left(\mathscr{E}_i + E_{0i} + \frac{1}{c} e_{ijk} v_j \mathscr{H}_k + \frac{1}{c} e_{ijk} v_j B_{0k}\right) + \frac{C_i}{m}, \tag{2.2}$$

where e and m are the charge and the mass of a particle, e_{ijk} is the Levi-Civita fully antisymmetric tensor and $\boldsymbol{C}$ is a friction force owing to collisions with other sorts of particles. To simplify the notation, we do not introduce here any symbol denoting the sort of particles.

According to Klíma (1967, 1968), the time-averaged terms in (2.2) arising from $\nabla \hat{E}^2$ and from $\boldsymbol{C}$ are proportional to $(k_0 a)^{-1}$ and to ν/ω, respectively, ν being some effective collision frequency. The basic assumption of the following analysis is that the values of $1/(k_0 a)$, ν/ω and $1/(\omega\tau)$ are small parameters. In terms of these parameters, we neglect the second- and higher-order contributions. Within this approximation, the time-averaged force can be written in the following form:

$$\boldsymbol{F} = \boldsymbol{F}_a + \boldsymbol{F}_\nu + \boldsymbol{F}_\tau, \tag{2.3}$$

where the individual terms on the right-hand side are proportional to $\hat{E}^2/(k_0 a)$, $\hat{E}^2 \nu/\omega$ and $\hat{E}^2/(\omega\tau)$, respectively. Since $\boldsymbol{F}_a$ and $\boldsymbol{F}_\nu$ are given (Klíma 1967, 1968), only $\boldsymbol{F}_\tau$ remains to be determined. Using equation (2.2) for this purpose, we can therefore ignore C_i and (where suitable) spatial dependences of $\hat{E}_i$, E_{0i} and B_{0k}. Consequently, the time average of (2.2) can be substituted by a space average. This way we avoid the rather complicated application (Klíma 1972, Milantiev 1976) of the Bogoliubov–Zubarev method.

3. Space average of the equation of motion

Applying the Fourier transform to (2.2), we obtain

$$\frac{dv_i(\boldsymbol{k})}{\partial t} = -\mathrm{i} \int \mathrm{d}^3\boldsymbol{k}'\, \mathrm{d}^3\boldsymbol{k}''\, \delta(\boldsymbol{k} - \boldsymbol{k}' - \boldsymbol{k}'') v_j(\boldsymbol{k}') k_j'' v_i(\boldsymbol{k}'') + \frac{e}{m}\bigg(E_i(\boldsymbol{k}) + E_{0i}(\boldsymbol{k})$$

$$+ \frac{e_{ijl}}{c} \int \mathrm{d}^3\boldsymbol{k}'\, \mathrm{d}^3\boldsymbol{k}''\, \delta(\boldsymbol{k} - \boldsymbol{k}' - \boldsymbol{k}'') v_j(\boldsymbol{k}') H_l(\boldsymbol{k}'') + \frac{e_{ijl}}{c} v_j(\boldsymbol{k}) B_{0l}\bigg), \tag{3.1}$$

where

$$E_i(\boldsymbol{k})=\frac{1}{(2\pi)^3}\int \mathrm{d}^3\boldsymbol{x}\, \mathrm{e}^{-\mathrm{i}k_jx_j}\mathscr{E}_i,$$
$$H_l(\boldsymbol{k})=\frac{1}{(2\pi)^3}\int \mathrm{d}^3\boldsymbol{x}\, \mathrm{e}^{-\mathrm{i}k_jx_j}\mathscr{H}_l, \tag{3.2}$$

etc, and C_i has been omitted according to § 2. The $\boldsymbol{k}=0$ component of the Fourier transform is proportional to the space-averaged value (see also § 5). Therefore, we put $\boldsymbol{k}\to 0$ in equation (3.1):

$$\frac{\partial v_i^0}{\partial t}=\mathrm{i}\int \mathrm{d}^3\boldsymbol{k}\, v_j(\boldsymbol{k})k_jv_i^*(\boldsymbol{k})+\frac{e}{m}\left(E_{0i}^0+\frac{e_{ijl}}{c}v_j^0B_{0l}+\frac{e_{ijl}}{c}\int \mathrm{d}^3\boldsymbol{k}\, v_j(\boldsymbol{k})H_l^*(\boldsymbol{k})\right), \tag{3.3}$$

where $(\ldots)^0\equiv(\ldots)_{\boldsymbol{k}\to 0}$. Since the first term on the right-hand side of (3.3) is quadratic in $\boldsymbol{v}(\boldsymbol{k})$, it is sufficient to determine $\boldsymbol{v}(\boldsymbol{k})$ in the linear approximation with respect to $\hat{\boldsymbol{E}}$. Omitting $\boldsymbol{E}_0(\boldsymbol{k})$ in the oscillating motion $\boldsymbol{v}(\boldsymbol{k})$, we have from (3.1)

$$\frac{\partial v_i(\boldsymbol{k})}{\partial t}=\frac{e}{m}\left(E_i(\boldsymbol{k})+\frac{e_{ijl}}{c}v_j(\boldsymbol{k})B_{0l}\right). \tag{3.4}$$

Inserting (2.1) into the first of relations (3.2), we obtain

$$E_i(\boldsymbol{k})=E_i^+(\boldsymbol{k})\,\mathrm{e}^{\mathrm{i}\omega t}+E_i^-(\boldsymbol{k})\,\mathrm{e}^{-\mathrm{i}\omega t}, \tag{3.5}$$

where

$$E_i^{\pm}=\frac{1}{2(2\pi)^3}\int \mathrm{d}^3\boldsymbol{x}\,\hat{E}_i(t,\boldsymbol{x})\exp[-\mathrm{i}(k_j\pm k_{0j})x_j\pm\mathrm{i}\alpha_i] \tag{3.6}$$

are functions of $\boldsymbol{k}$ and slowly varying functions of t. Then, $v_i(\boldsymbol{k})$ can be split into

$$v_i(\boldsymbol{k})=v_i^++v_i^- \tag{3.7}$$

so that (3.4) gives

$$\frac{\partial v_i^{\pm}}{\partial t}=\frac{e}{m}\left(E_i^{\pm}\ \mathrm{e}^{\pm\mathrm{i}\omega t}+\frac{e_{ijl}}{c}v_j^{\pm}B_{0l}\right). \tag{3.7}$$

To find v_i^-, we put

$$v_i^-=(u_i^-+w_i^-)\,\mathrm{e}^{-\mathrm{i}\omega t}, \tag{3.9}$$

where u_i^- fulfills the following algebraic equation:

$$-\mathrm{i}\omega u_i^-=\frac{e}{m}\left(E_i^-+\frac{e_{ijl}}{c}u_j^-B_{0l}\right).$$

Consequently, we can write

$$u_i^-=M_{il}E_l^-,\qquad M_{il}=\sigma_{il}/en_0, \tag{3.10}$$

where σ_{il} is the well known conductivity tensor for the sort of particles in question, and n_0 is their averaged concentration. Explicitly,

$$M_{ij}=\frac{e}{m}\frac{\omega}{\omega^2-\Omega^2}\left(\mathrm{i}\delta_{ij}-\mathrm{i}\frac{\Omega_i\Omega_j}{\omega^2}-e_{ijl}\frac{\Omega_l}{\omega}\right) \tag{3.11}$$

with $\Omega_i = eB_{0i}/(mc)$, $\Omega^2 = \Omega_j\Omega_j$. The substitution of (3.9) into (3.8) yields

$$w_i^- = -\frac{m}{e} M_{in} M_{nl} \frac{\partial E_l^-}{\partial t}. \tag{3.12}$$

The value of v_i^+ is found quite similarly and (3.7) then is

$$v_i(\boldsymbol{k}) = e^{i\omega t}\left(M_{il}^* E_l^+ - \frac{m}{e} M_{in}^* M_{nl}^* \frac{\partial E_l^+}{\partial t}\right) + e^{-i\omega t}\left(M_{il} E_l^- - \frac{m}{e} M_{in} M_{nl} \frac{\partial E_l^-}{\partial t}\right). \tag{3.13}$$

We note that $M_{il}^* = -M_{li}$ and $E_i^-(\boldsymbol{k}) = E_i^{+*}(-\boldsymbol{k})$.

To express the last term inside the large parentheses in (3.3) in terms of $\boldsymbol{E}^{\pm}$, the Maxwell equation

$$\partial H_l(\boldsymbol{k})/\partial t = -\mathrm{i}c e_{lmn} k_m E_n(\boldsymbol{k}) \tag{3.14}$$

is used. By the same procedure as used between (3.4) and (3.13), we obtain

$$H_l(\boldsymbol{k}) = -e^{i\omega t}\frac{c}{\omega} e_{lmn} k_m\left(E_n^+ + \frac{\mathrm{i}}{\omega}\frac{\partial E_n^+}{\partial t}\right) + e^{-i\omega t}\frac{c}{\omega} e_{lmn} k_m\left(E_n^- - \frac{\mathrm{i}}{\omega}\frac{\partial E_n^-}{\partial t}\right). \tag{3.15}$$

The expressions (3.13) and (3.15) are substituted into (3.3):

$$\begin{aligned}\frac{\partial v_i^0}{\partial t} = {} & \frac{e}{m}\left(E_{0i}^0 + \frac{e_{ijl}}{c} v_j^0 B_{0l}\right) + \int \mathrm{d}^3\boldsymbol{k}\, k_j\Big[\mathrm{i} M_{jr}^* M_{il} E_r^+ E_l^{+*} \\ & + \mathrm{i}\frac{m}{e}\left(M_{jr} M_{in}^* M_{nl}^* E_r^{+*} \frac{\partial E_l^+}{\partial t} + M_{il}^* M_{js} M_{sr} E_l^+ \frac{\partial E_r^{+*}}{\partial t}\right) + \mathrm{cc}\Big] \\ & + \frac{e}{m\omega} e_{ijk} \int \mathrm{d}^3\boldsymbol{k}\Big[-M_{jr} E_r^{+*} e_{klm} k_l E_m^+ - \frac{\mathrm{i}}{\omega} M_{jr} E_r^{+*} e_{klm} k_l \frac{\partial E_m^+}{\partial t} \\ & + \frac{m}{e} M_{js} M_{sr} \frac{\partial E_r^{+*}}{\partial t} e_{klm} k_l E_m^+ + \mathrm{cc}\Big],\end{aligned} \tag{3.16}$$

where cc stands for complex conjugate. This equation is transformed and simplified in § 4.

4. Averaging of velocity along the oscillating trajectory

Similarly to Klíma (1967, 1968), it is convenient to introduce velocity $\boldsymbol{V}$:

$$V_i = v_i + \xi_j \frac{\partial v_i}{\partial x_j}, \tag{4.1}$$

where ξ_j is defined by

$$\partial \xi_j/\partial t = v_j. \tag{4.2}$$

In other words, $\boldsymbol{\xi}$ is the oscillating displacement of the fluid volume element. From (4.1)

$$V_i^0 = v_i^0 - \mathrm{i}\int \mathrm{d}^3\boldsymbol{k}\, \xi_j(\boldsymbol{k}) k_j v_i^*(\boldsymbol{k}), \tag{4.3}$$

where $\xi_j(\boldsymbol{k})$ is calculated from Fourier transformed equation (4.2) in the same approximation as used below (3.4):

$$\xi_j(\boldsymbol{k}) = \mathrm{e}^{\mathrm{i}\omega t}\left(-\frac{\mathrm{i}}{\omega}M^*_{jr}E^+_r + \frac{\mathrm{i}m}{e\omega}M^*_{js}M^*_{sr}\frac{\partial E^+_r}{\partial t} + \frac{1}{\omega^2}M^*_{jr}\frac{\partial E^+_r}{\partial t}\right)$$
$$+\,\mathrm{e}^{-\mathrm{i}\omega t}\left(\frac{\mathrm{i}}{\omega}M_{jr}E^-_r - \frac{\mathrm{i}m}{e\omega}M_{js}M_{sr}\frac{\partial E^-_r}{\partial t} + \frac{1}{\omega^2}M_{jr}\frac{\partial E^-_r}{\partial t}\right). \tag{4.4}$$

Inserting (4.3) with (4.4) into (3.16) gives

$$\frac{\partial V^0_i}{\partial t} = \frac{e}{m}E^0_{0i} + e_{ijk}V^0_j\,\Omega_k + 2\,\mathrm{Re}\int \mathrm{d}^3\boldsymbol{k}\left[-\frac{k_j}{\omega}M_{jr}M^*_{il}\frac{\partial E^{+*}_r E^+_l}{\partial t}\right.$$
$$+\,\mathrm{i}\frac{m}{e}k_j\left(M_{jr}M^*_{in}M^*_{nl}E^{+*}_r\frac{\partial E^+_l}{\partial t} + M^*_{il}M_{js}M_{sr}E^+_l\frac{\partial E^{+*}_r}{\partial t}\right)$$
$$-\,e_{imk}\frac{m\Omega_k}{e\omega}k_j\left(M_{jr}E^{+*}_r M^*_{mn}M^*_{nl}\frac{\partial E^+_l}{\partial t} + M_{js}M_{sr}\frac{\partial E^{+*}_r}{\partial t}M^*_{ml}E^+_l\right)$$
$$-\,e_{imk}\frac{\Omega_k}{\omega^2}\mathrm{i}k_jM_{jr}\frac{\partial E^{+*}_r}{\partial t}M^*_{ml}E^+_l$$
$$-\,\frac{\mathrm{i}e}{m\omega^2}k_iM_{jr}E^{+*}_r\frac{\partial E^+_j}{\partial t} + \frac{k_i}{\omega}M_{js}M_{sr}\frac{\partial E^{+*}_r}{\partial t}E^+_j$$
$$\left.+\,\frac{\mathrm{i}e}{m\omega^2}k_jM_{jr}E^{+*}_r\frac{\partial E^+_i}{\partial t} - \frac{k_j}{\omega}M_{js}M_{sr}\frac{\partial E^{+*}_r}{\partial t}E^+_i\right]. \tag{4.5}$$

In contrast to (3.16), only terms proportional to time derivatives of the oscillating field amplitude remain in the last integral. It follows immediately from (3.11) that

$$M_{in}M_{nl} \equiv -\frac{\mathrm{i}e}{m}\frac{\partial M_{il}}{\partial\omega} \tag{4.6}$$

and

$$\mathrm{i}M_{il} + e_{imk}\frac{\Omega_k}{\omega}M_{ml} + \frac{e}{m\omega}\delta_{il} \equiv 0. \tag{4.7}$$

Multiplying this identity by $k_j\omega$ and differentiating it yields

$$-\mathrm{i}k_jM^*_{il} - \mathrm{i}k_j\omega\frac{\partial M^*_{il}}{\partial\omega} + e_{imk}\Omega_k k_j\frac{\partial M^*_{ml}}{\partial\omega} \equiv 0. \tag{4.8}$$

The following relation is obtained by multiplying (4.7) by $k_j\omega M_{jr}$, differentiating it and using (4.8):

$$k_j\frac{\partial M_{jr}}{\partial\omega}M^*_{il} + \frac{\mathrm{i}}{\omega}e_{imk}\Omega_k k_j\frac{\partial M_{jr}}{\partial\omega}M^*_{ml} + \frac{\mathrm{i}e}{m\omega}k_j\delta_{il}\frac{\partial M_{jr}}{\partial\omega} \equiv 0. \tag{4.9}$$

The identities (4.6)–(4.9) simplify equation (4.5) considerably:

$$\frac{\partial V^0_i}{\partial t} = \frac{e}{m}E^0_{0i} + e_{ijk}V^0_j\,\Omega_k + \frac{2e}{m\omega^2}\,\mathrm{Re}\int \mathrm{d}^3\boldsymbol{k}\left(\mathrm{i}k_jM_{jr}\frac{\partial E^+_i E^{+*}_r}{\partial t} - \mathrm{i}k_iE^{+*}_r\frac{\partial E^+_j}{\partial t}\frac{\partial\omega M_{jr}}{\partial\omega}\right). \tag{4.10}$$

5. Final equation and particle fluxes

Consider a flat wave packet which is very long in all three coordinates (cube L_p^3). Then it is easy to find from (3.6) that

$$\int \mathrm{d}^3\boldsymbol{k}\; k_l E_j^+ E_r^{+*} = \frac{k_{0l}}{4}\left(\frac{L_\mathrm{p}}{2\pi}\right)^3 \hat{E}_j(t)\hat{E}_r(t)\exp[\mathrm{i}(\alpha_j-\alpha_r)]. \tag{5.1}$$

Instead of $\mathscr{E}$ (2.1), we now introduce the more usual complex representation

$$E_j = \hat{E}_j \exp[-\mathrm{i}(\omega t - k_{0l}x_l + \alpha_j)] \tag{5.2}$$

so that $\mathscr{E}_j = \mathrm{Re}\,(E_j)$ and, from (5.1),

$$\int \mathrm{d}^3\boldsymbol{k}\; k_l E_j^+ E_r^{+*} = \frac{k_{0l}}{4}\left(\frac{L_\mathrm{p}}{2\pi}\right)^3 E_j^* E_r. \tag{5.3}$$

Further, we define the space average symbol $\langle\ldots\rangle$, i.e.

$$\langle V_j\rangle = \left(\frac{1}{L_\mathrm{p}}\right)^3 \int \mathrm{d}^3\boldsymbol{x}\; V_j. \tag{5.4}$$

Then we have

$$V_j^0 = \left(\frac{L_\mathrm{p}}{2\pi}\right)^3 \langle V_j\rangle. \tag{5.5}$$

This space averaging is obviously equivalent to averaging over quickly varying phases, see the argument of the cosine in (2.1). Consequently, $\langle \boldsymbol{V}\rangle$ is identical with $\boldsymbol{V}_\mathrm{L}$ used in (Klíma 1967, 1968). Equation (4.10) then is

$$\frac{\partial V_{\mathrm{L}i}}{\partial t} = \frac{e}{m}E_{0i} + e_{ijk}V_{\mathrm{L}j}\Omega_k + \frac{F_{\tau i}}{m} \tag{5.6}$$

with

$$F_{\tau i} = \frac{\mathrm{i}e}{4\omega^2}\left(-k_{0j}M_{jr}\frac{\partial E_i^* E_r}{\partial t} + k_{0j}M_{jr}^*\frac{\partial E_i E_r^*}{\partial t} + k_{0i}\frac{\partial \omega M_{jr}}{\partial \omega}\frac{\partial E_j^* E_r}{\partial t}\right). \tag{5.7}$$

Let us consider the more general case when $\nu \neq 0$, $\boldsymbol{B}_0$, $\boldsymbol{E}_0$ and the oscillating field amplitudes depend on $\boldsymbol{x}$, too. Then the time-averaged motion is governed by equation (3.10) in Klíma (1968) with the right-hand side supplemented by $\boldsymbol{F}_\tau$ (5.7). Time dependences of field amplitudes have been admitted in the paper just referred to. In fact, consequent effects have not been considered either there or in Klíma (1967). We note that the previous results (Klíma 1967, 1968) are valid for arbitrary ν/ω. For simplicity, we present here explicitly only the case $\nu = 0$ which is needed in § 6:

$$n_0 m\left(\frac{\partial V_{\mathrm{L}i}}{\partial t} + V_{\mathrm{L}j}\frac{\partial V_{\mathrm{L}i}}{\partial x_j}\right) = en_0E_{0i} + \frac{e}{c}n_0e_{ijk}V_{\mathrm{L}j}B_{0k} - n_0\frac{\partial W}{\partial x_i} + n_0F_{\tau i}, \tag{5.8}$$

where $F_{\tau i}$ is given by (5.7) and

$$W = \frac{e^2}{4m\omega^2}\left(1-\frac{\Omega^2}{\omega^2}\right)^{-1}\left(E_j^*E_j - \frac{1}{\omega^2}E_j^*\Omega_jE_l\Omega_l + \frac{\mathrm{i}}{\omega}\Omega_je_{jkl}E_k^*E_l\right). \tag{5.9}$$

To derive the partial current density $j_i = env_i$, we again first omit spatial dependence of $\boldsymbol{B}_0$. We put $n = n_0 + \tilde{n}$ and assume for the moment that n_0 is also homogeneous, $\tilde{n}$ being the oscillating part of the concentration. The $\boldsymbol{k}=0$ component of the Fourier transform of j_i is

$$j_i^0 = en_0 v_i^0 + e\int \mathrm{d}^3\boldsymbol{k}\, \tilde{n}(\boldsymbol{k}) v_i^*(\boldsymbol{k}). \tag{5.10}$$

From the continuity equation,

$$\partial \tilde{n}(\boldsymbol{k})/\partial t = -\mathrm{i} n_0 k_j v_j(\boldsymbol{k}). \tag{5.11}$$

We easily find by comparing (5.11) with (4.2) that $\tilde{n}(\boldsymbol{k}) = -\mathrm{i} n_0 k_j \xi_j$. In view of (4.3), equation (5.10) then simplifies to $j_i^0 = en_0 V_i^0$. Consequently, the partial current density averaged at a fixed point of space is $j_{0i} = en_0 V_{\mathrm{L}i}$. If space dependences of fields are included, this result is to be supplemented by another term derived previously (Klíma 1967, 1968), i.e.

$$j_{0i} = en_0 V_{\mathrm{L}i} + c e_{ijk} \frac{\partial \mathcal{M}_k}{\partial x_j}, \tag{5.12}$$

where

$$\mathcal{M}_k = \frac{1}{4\omega} E_l^* E_m \frac{\partial \sigma_{lm}}{\partial B_{0k}} \tag{5.13}$$

is the magnetic moment density of the partial oscillating current.

6. Discussion and conclusion

With proper values of e and m, equation (5.8) is valid both for electrons and ions. The sum of those equations can be transformed in a similar way to that done in Klíma and Petržílka (1968). However, the transformation method is modified by including the time dependence of field amplitudes in Maxwell's equations. The result is (see appendix)

$$n_0 \sum_{\mathrm{e,i}} m\left(\frac{\partial V_{\mathrm{L}j}}{\partial t} + V_{\mathrm{L}k}\frac{\partial V_{\mathrm{L}j}}{\partial x_k}\right) = \frac{\partial T_{jk}}{\partial x_k} - \frac{\partial g_j}{\partial t}, \tag{6.1}$$

where T_{jk} is the tensor of time-averaged stresses (Klíma and Petržílka 1968):

$$T_{jk} = -\frac{\delta_{jk}}{16\pi}\left(2B_0^2 + E_l^* E_l + H_l^* H_l - B_{0l} E_i^* E_m \frac{\partial \epsilon_{im}}{\partial B_{0l}}\right)$$
$$+\frac{1}{16\pi}\left(4B_{0j}B_{0k} + H_j^* H_k + H_j H_k^* - B_{0j} E_i^* E_m \frac{\partial \epsilon_{im}}{\partial B_{0k}} + \epsilon_{jl} E_l E_k^* + \epsilon_{lj} E_l^* E_k\right), \tag{6.2}$$

$\epsilon_{jl} = \delta_{jl} + 4\pi \mathrm{i} \Sigma_{\mathrm{i,e}}\, \sigma_{jl}/\omega$, $\boldsymbol{H}$ corresponds to $\boldsymbol{\mathcal{H}}$ in complex representation (5.2) and

$$g_i = \frac{1}{8\pi c} \mathrm{Re}(e_{jkl} E_k H_l^*) \tag{6.3}$$

is the average field momentum density. Equation (6.1) coincides with the *ansatz* used by Washimi and Karpman (1976, their equation (20)). It has been pointed out by de Groot and Suttorp (1967, 1968) that such relations must be derived from microscopic considerations. We do not intend to discuss this problem in detail and refer to Brevik (1970), Shearer and Eddleman (1973), Klíma and Petržílka (1974, 1975a,b), Ginzburg and Ugarov (1976), Skobeltsyn (1977) and Ginzburg (1977). Washimi and Karpman (1976) also give briefly an alternative (microscopic) derivation of the time-averaged force. However, their final results (21) and/or (28) both differ from our formula (5.7) (summed over electrons and ions) by

$$-\frac{k_n}{16\pi}\left(\frac{\partial \epsilon_{nl}}{\partial \omega}E_i^*\frac{\partial E_l}{\partial t}+\mathrm{cc}\right), \tag{6.4}$$

which does not vanish in general. Since Washimi and Karpman (1976) do not present the relevant algebra in detail, we were not able to trace the origin of that difference. This is also the reason why we give the transition from (5.8) to (6.1) in the appendix.

On the other hand, our formula (5.7) is consistent with Washimi (1973) and Milantiev (1976), who considered waves propagating along $\boldsymbol{B}_0$. It is interesting to note that the form of (5.7) coincides with the $\partial \hat{E}^2/\partial t$-terms in Jungwirth (1972) derived for anisotropic but non-magnetised plasma.

The results of the present paper generalise the hydrodynamic analysis of Klíma (1967, 1968) by including the time dependence of the oscillating field amplitudes. While new terms proportional to $\partial \hat{E}^2/\partial t$ arise in the equation of motion, the averaged current density $\boldsymbol{j}_0$ depends on time only implicitly (5.12). In the approximation used here, ν/ω is a small parameter (§ 2). Consequently, if collisions are included ($\nu \neq 0$), ν does not enter the quasi-potential W and (5.9) is also valid.

Appendix

In the following, the transition from (5.8) to (6.1) is given. Since $\boldsymbol{E}_0$ yields only standard contributions, we put $\boldsymbol{E}_0=0$ for simplicity. The algebra (3.1)–(3.7) in Klíma and Petržílka (1968) can be used here immediately. Therefore, the expression to be transformed is

$$-\frac{E_j^*E_k}{16\pi}\frac{\partial \epsilon_{jk}}{\partial x_m}=-\frac{1}{16\pi}\frac{\partial}{\partial x_m}(E_j^*E_k\epsilon_{jk})+\frac{\epsilon_{jk}}{16\pi}\left(E_k\frac{\partial E_j^*}{\partial x_m}+E_j^*\frac{\partial E_k}{\partial x_m}\right). \tag{A.1}$$

From the equation $\partial D_k/\partial t=4\pi\,\Sigma_{\mathrm{e,i}}\, j_k+\partial E_k/\partial t$, we find

$$D_k=\epsilon_{kj}E_j+\mathrm{i}\frac{\partial \epsilon_{kj}}{\partial \omega}\frac{\delta E_j}{\delta t}, \tag{A.2}$$

where $\delta/\delta t$ does not apply to $\exp(-\mathrm{i}\omega t)$. Equations (A.2), (3.14), $\nabla\cdot\boldsymbol{H}=0$ and

$$\nabla\times\boldsymbol{H}=-\frac{\mathrm{i}\omega}{c}\boldsymbol{D}+\frac{1}{c}\frac{\delta \boldsymbol{D}}{\delta t} \tag{A.3}$$

are used here instead of (3.8) and (3.10) in Klíma and Petržílka (1968). The last term in (A.1) then is

$$\begin{aligned}\frac{\epsilon_{jk}}{16\pi}&\left(E_k\frac{\partial E_j^*}{\partial x_m}+E_j^*\frac{\partial E_k}{\partial x_m}\right)\\ &=\frac{1}{16\pi}\left[\frac{\partial}{\partial x_j}(D_jE_m^*-H_j^*H_m)-\frac{1}{2}\frac{\partial}{\partial x_m}(H_jH_j^*)\right.\\ &\quad\left.+\frac{1}{c}e_{mjk}\left(H_j^*\frac{\delta D_k}{\delta t}+D_k\frac{\delta H_j^*}{\delta t}\right)+\mathrm{i}\frac{\partial E_k}{\partial x_m}\frac{\partial \epsilon_{jk}}{\partial \omega}\frac{\delta E_j^*}{\delta t}\right]+\mathrm{cc}.\end{aligned} \tag{A.4}$$

According to (5.8), the force density f_m (3.6) in Klíma and Petržílka (1968) is to be supplemented by $n_0 F_{\tau m}$ (5.7) summed over electrons and ions:

$$f_m = \frac{\partial T_{nn}}{\partial x_n} - \frac{e_{mjk}}{16\pi c}\frac{\partial}{\partial t}(E_j H_k^* + \mathrm{cc}) + n_0 \sum_{\mathrm{e,i}} F_{\tau m}$$
$$+ \frac{1}{16\pi}\left(\frac{e_{mjk}}{c}\frac{\partial}{\partial t}[(E_j - D_j)H_k^*] - k_m E_k \frac{\partial \epsilon_{jk}}{\partial \omega}\frac{\delta E_j^*}{\delta t} + \mathrm{cc}\right). \quad \text{(A.5)}$$

By using (3.14), the last two terms cancel out and (6.1) is established.

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